New Ideals in Classical Iwasawa Theory

David Solomon, King's College London, United Kingdom

Let K be an absolutely abelian field and $\Lambda = \mathbb{Z}_p[[\mathrm{Gal}(K(\mu_{p^\infty})/\mathbb{Q})]]$ for $p \neq 2$. I shall explain the construction of a certain Λ-module homomorphism $\mathfrak{j}_\infty : \mathfrak{X}_\infty \to \Lambda$ where $\mathfrak{X}_\infty$ is the Galois group of the maximal abelian pro-p extension of $K(\mu_{p^\infty})$ unramified outside p. I shall then describe some applications of $\mathfrak{j}_\infty$ including:

(i). a new analogue of Stickelberger's Theorem in the plus part,

(ii). a new variant 'at $s = 1$' of the usual Stickelberger ideal in the minus part and

(iii). some new exact sequences relating to the classical Main Conjecture over $\mathbb{Q}$.

The tools involve explicit reciprocity laws and a new interpretation of Thaine's method. If time permits, I shall discuss possible generalisations and perspectives, including a link with the so-called 'Congruence Conjecture' for Rubin-Stark elements.